

Oscillations and the Simple Harmonic Motion (SHM) Model

(Review of material treated in Unit 7 of Physics 7B)

Introduction

Many physical phenomena exhibit “vibrations.” The everyday use of the word “vibrate” to mean that something moves back and forth, usually in a regular way, is very close to what is meant by “vibration” or “oscillation” in physics. When we examine objects that are oscillating, we notice several features. The motion repeats in a cyclic manner. As the object moves toward one extreme, a net force acts on the object, causing it to stop and move towards the opposite extreme. In order to make these ideas more precise we introduce a model of a very common kind of oscillation, called simple harmonic motion.

Some oscillations occur with abrupt changes in an object’s motion. An example is a vertically bouncing ping-pong ball. Going down, the ball changes direction very rapidly when it hits the table. Going up, it gradually comes to a stop and then gradually begins moving down. In contrast, a swinging pendulum changes its direction at both ends of its swing in a very smooth way. This latter, very smooth oscillatory motion is called simple harmonic motion. Because this motion is so common, it is useful to formulate a simple harmonic motion (SHM) model. This model will become the starting point of a wave model taken up in Unit 8.

The essential elements of the SHM model (as well as all other models) include **constructs** (the ideas that form the building blocks of the model), the **relationships** between the constructs, and the **representations** that allow us to readily use the model to develop explanations of various phenomena, make precise predictions regarding oscillatory motion, and, in general, to use the model to make sense of a vast variety of oscillatory phenomena. What follows is a succinct listing of the constructs, relationships, and representations of the SHM model. Remember, the constructs are the ideas that everyone should have a common understanding of in order to communicate with one another and to make sense of the relationships among the constructs. Some constructs are nothing more than definitions. Others are more difficult to get hold of. Regardless, the relationships and representations will make little sense, if you don’t have a solid understanding of the individual constructs. Listed below are the constructs, relationships and representations of the SHM model, which form the foundation for understanding waves and the plane wave model introduced in Unit 8.

Constructs, Relationships, and Representations of the SHM Model

Equilibrium (position)

construct

The stable position of an object for which the sum of the forces (net force) acting on the object is zero (cancel each other out). An oscillating object will “end up” at the equilibrium position if there is dissipation (transfer of mechanical energy to thermal energy systems). (There is always dissipation associated with macroscopic oscillators, but not necessarily with microscopic oscillators.)

Displacement

construct

The change in position of an object measured from its equilibrium position. When an object oscillates it moves alternately in a positive and negative direction about its equilibrium position; the displacement alternates from being positive to being negative.

Restoring Force

construct

A net force acting on an object that is displaced from its equilibrium position that tends to cause it to move back towards its equilibrium position.

Simple harmonic motion (SHM)

relationship

A particularly common type of oscillatory motion that results when the magnitude of the restoring force is directly proportional to the displacement of the object from its equilibrium position. That is, $\Sigma F = -ky$, where k is a constant and y is the value of the position variable measured from the equilibrium position. For most oscillating physical systems, the restoring force becomes linear in displacement for sufficiently small oscillations. Thus, most physical systems oscillate in SHM for sufficiently small oscillations.

Amplitude

construct

The amplitude of an object oscillating in SHM is the absolute value of the maximum displacement. Amplitude is controlled by the magnitude of the forces that start the oscillation. Amplitude is often denoted by the symbol A .

Period (and frequency)

construct

Period is the time it takes for an oscillator to make one complete cycle (represented as T). The SI unit of period is second (s).

The frequency, represented as f , is the number of oscillations occurring in one unit of time. Frequency is the reciprocal of the period: $f = 1/T$. The SI unit of frequency is reciprocal seconds ($1/s$), which is given the name hertz (Hz).

SHM varies sinusoidally in time

relationship

The position of an object oscillating in SHM varies sinusoidally in time, t . The argument of the sine and/or cosine function includes $2\pi t/T$

Fixed phase constant

construct

The fixed phase constant is a mathematical constant expressed as an angle that gives the proper value of the argument of a sine or cosine function used to describe SHM with respect to the particular initial conditions. The fixed phase constant is usually represented by ϕ (lower case “phi”).

Using a single sine function to describe an object oscillating in SHM

relationship

By including the phase constant in the argument of the sine function, $\sin(2\pi t/T + \varphi)$, we can make the sine function fit any particular physical situation. Without the phase constant, and with only a sine function, we would always have to start timing the oscillation when the position of the object, y , had the value zero and was just starting to increase. By including the phase constant, we can have a perfectly general solution for any set of initial conditions, while still using only the sine function instead of a combination of sine and cosine functions.

Mathematical representation of the motion of an object vibrating in SHM

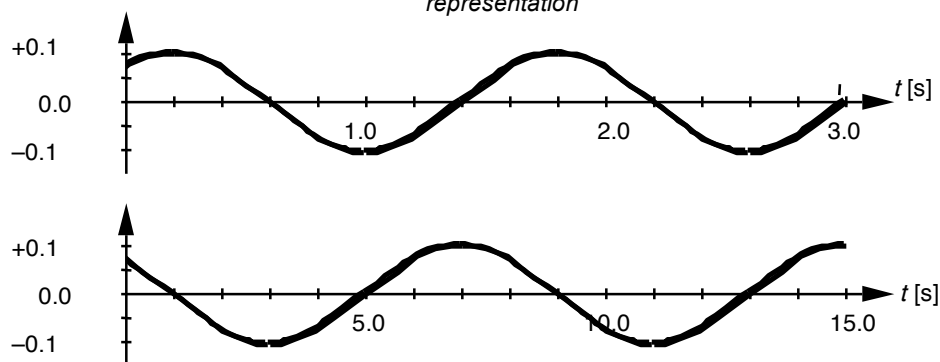
representation

Any object oscillating in SHM in one dimension can be represented mathematically with the following equation. The function $y(t)$ represents the displacement of the object from its equilibrium position using an appropriate coordinate. The value of the variable y at equilibrium is zero and it takes on positive and negative values as the object oscillates. The variable y can represent a distance, e.g., a mass oscillating on a spring, or an angle, e.g., a swinging pendulum.

$$y(t) = A\sin(2\pi t/T + \varphi)$$

Graphical representation of the motion of an object vibrating in SHM

representation



The top graph represents an object oscillating in SHM with a period, T , of 1.6 s, an amplitude, A , of 1.2 units, and a phase constant, φ , of about $\pi/4$ rad in the mathematical representation $y(t) = A\sin(2\pi t/T + \varphi)$. The lower graph has the same amplitude, but $T = 8$ s and $\varphi = 3/4 \pi$.

The most general form of the equation that describes *any* object undergoing SHM (simple harmonic motion) is given by:

$$y(t) = A \sin\left(\frac{2\pi t}{T} + \varphi\right) + B.$$

